Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Standard level
Paper 2

Wednesday 11 May 2016 (morning)

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics SL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[90 marks]**.

12 pages

2216 – 7306

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, for example if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.

1. [Maximum mark: 6]

The first three terms of an arithmetic sequence are $u_1 = 0.3$, $u_2 = 1.5$, $u_3 = 2.7$.

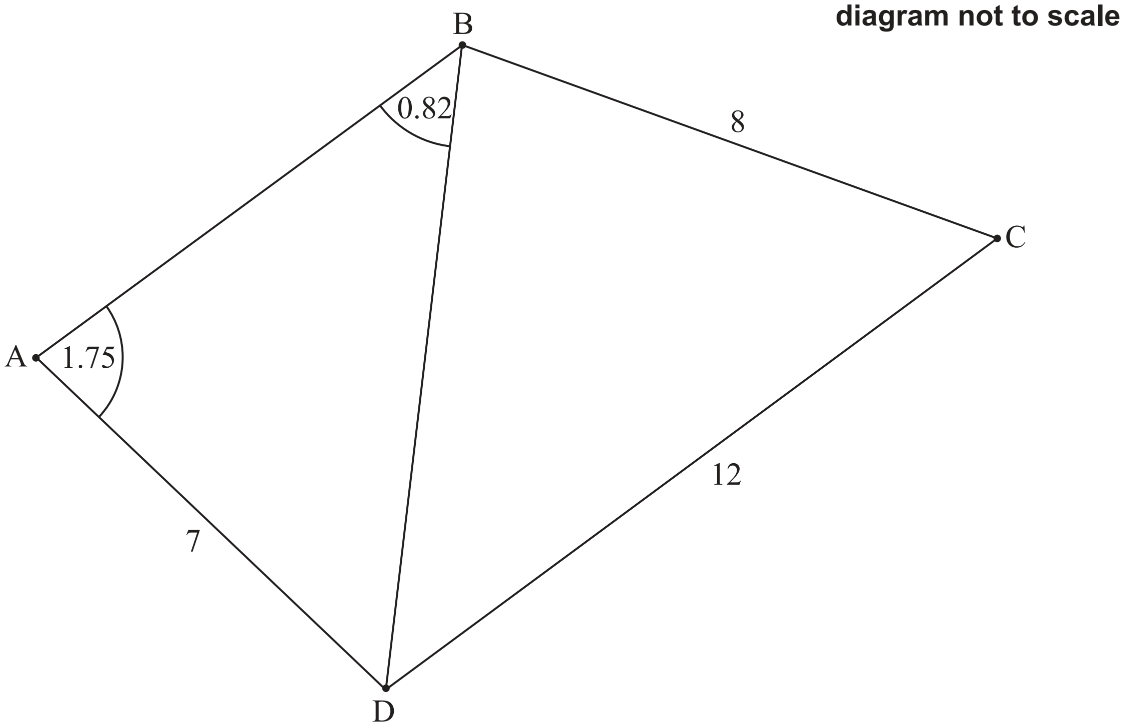
(a) Find the common difference. [2]

(b) Find the 30th term of the sequence. [2]

(c) Find the sum of the first 30 terms. [2]

2. [Maximum mark: 6]

The following diagram shows a quadrilateral ABCD.



$AD = 7\,\text{cm}$, $BC = 8\,\text{cm}$, $CD = 12\,\text{cm}$, $D\hat{A}B = 1.75$ radians, $A\hat{B}D = 0.82$ radians.

(a) Find BD. [3]

(b) Find $D\hat{B}C$. [3]

. . . .

3. [Maximum mark: 7]

Let $f(x) = e^{0.5x} - 2$.

(a) For the graph of f

(i) write down the y-intercept;

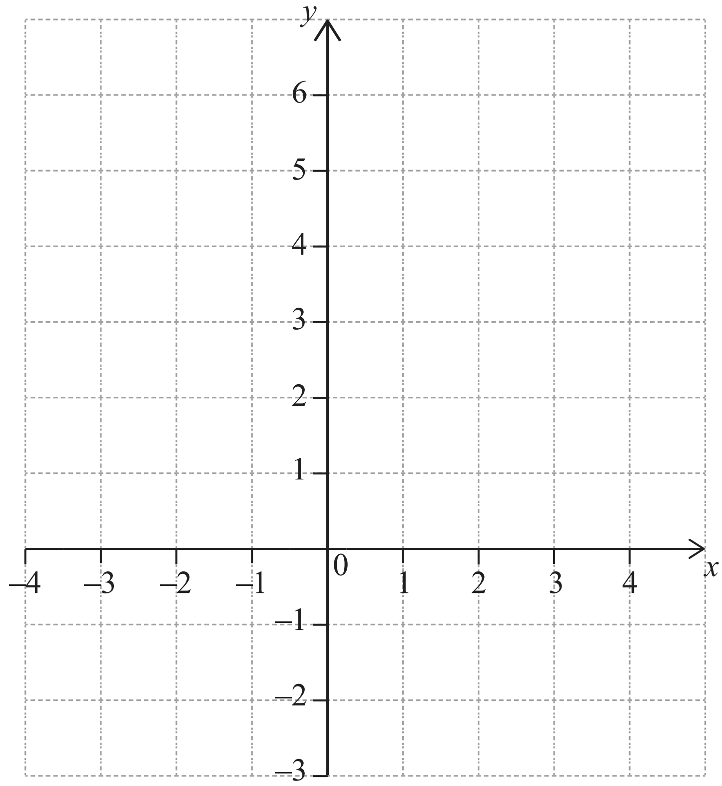
(ii) find the x-intercept;

(iii) write down the equation of the horizontal asymptote. [4]

(This question continues on the following page)

(Question 3 continued)

(b) On the following grid, sketch the graph of f, for $-4 \leq x \leq 4$. [3]



4. [Maximum mark: 8]

The height, h metres, of a seat on a Ferris wheel after t minutes is given by

$$h(t) = -15\cos 1.2t + 17, \text{ for } t \geq 0.$$

(a) Find the height of the seat when $t = 0$. [2]

(b) The seat first reaches a height of 20 m after k minutes. Find k. [3]

(c) Calculate the time needed for the seat to complete a full rotation, giving your answer correct to one decimal place. [3]

5. [Maximum mark: 6]

Consider the expansion of $\left(x^2+\frac{2}{x}\right)^{10}$.

(a) Write down the number of terms of this expansion. [1]

(b) Find the coefficient of x^8. [5]

6. [Maximum mark: 6]

A competition consists of two independent events, shooting at 100 targets and running for one hour.

The number of targets a contestant hits is the S score. The S scores are normally distributed with mean 65 and standard deviation 10.

(a) A contestant is chosen at random. Find the probability that their S score is less than 50. [2]

The distance in km that a contestant runs in one hour is the R score. The R scores are normally distributed with mean 12 and standard deviation 2.5. The R score is independent of the S score.

Contestants are disqualified if their S score is less than 50 **and** their R score is less than $x\ \text{km}$.

(b) Given that 1% of the contestants are disqualified, find the value of x. [4]

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7. [Maximum mark: 7]

A particle moves in a straight line. Its velocity $v\,\mathrm{m\,s^{-1}}$ after t seconds is given by

$$v = 6t - 6, \text{ for } 0 \le t \le 2.$$

After p seconds, the particle is $2\,\mathrm{m}$ from its initial position. Find the possible values of p.

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 15]

The price of a used car depends partly on the distance it has travelled. The following table shows the distance and the price for seven cars on 1 January 2010.

Distance, x km	11 500	7500	13 600	10 800	9500	12 200	10 400
Price, y dollars	15 000	21 500	12 000	16 000	19 000	14 500	17 000

The relationship between x and y can be modelled by the regression equation $y = ax + b$.

(a) (i) Find the correlation coefficient.

(ii) Write down the value of a and of b. [4]

On 1 January 2010, Lina buys a car which has travelled 11 000 km.

(b) Use the regression equation to estimate the price of Lina's car, giving your answer to the nearest 100 dollars. [3]

The price of a car decreases by 5 % each year.

(c) Calculate the price of Lina's car after 6 years. [4]

Lina will sell her car when its price reaches 10 000 dollars.

(d) Find the year when Lina sells her car. [4]

Do **not** write solutions on this page.

9. [Maximum mark: 14]

Let $f(x)=\frac{1}{x-1}+2$, for $x>1$.

(a) Write down the equation of the horizontal asymptote of the graph of f. [2]

(b) Find $f'(x)$. [2]

Let $g(x)=a\mathrm{e}^{-x}+b$, for $x\geq 1$. The graphs of f and g have the same horizontal asymptote.

(c) Write down the value of b. [2]

(d) Given that $g'(1)=-\mathrm{e}$, find the value of a. [4]

(e) There is a value of x, for $1<x<4$, for which the graphs of f and g have the same gradient. Find this gradient. [4]

Do **not** write solutions on this page.

10. [Maximum mark: 15]

Consider the points $A(1, 5, -7)$ and $B(-9, 9, -6)$.

(a) Find $\overrightarrow{AB}$. [2]

Let C be a point such that $\overrightarrow{AC} = \begin{pmatrix} 6 \\ -4 \\ 0 \end{pmatrix}$.

(b) Find the coordinates of C. [2]

The line L passes through B and is parallel to (AC).

(c) Write down a vector equation for L. [2]

(d) Given that $\left|\overrightarrow{AB}\right| = k\left|\overrightarrow{AC}\right|$, find k. [3]

(e) The point D lies on L such that $\left|\overrightarrow{AB}\right| = \left|\overrightarrow{BD}\right|$. Find the possible coordinates of D. [6]